The signature of dark energy perturbations in galaxy cluster surveys

L. Raul Abramo* and Ronaldo C. Batista

Instituto de Física, Universidade de São Paulo, CP 66318, 05315-970, São Paulo, Brazil

Rogério Rosenfeld

Instituto de Física Teórica, Universidade Estadual Paulista,
R. Pamplona 145, 01405-900, São Paulo, Brazil

All models of dynamical dark energy possess fluctuations, which affect the number of galaxy clusters in the Universe. We have studied the impact of dark energy clustering on the number of clusters using a generalization of the spherical collapse model and the Press-Schechter formalism. Our statistical analysis is performed in a 7-parameter space using the Fisher matrix method, for several hypothetical Sunyaev-Zel'dovich and weak lensing (shear maps) surveys. In some scenarios, the impact of these fluctuations is large enough that their effect could already be detected by existing instruments such as the South Pole Telescope, when its data is combined with WMAP and SDSS. Future observations could go much further and probe the nature of dark energy by distinguishing between different models on the basis of their perturbations, not only their expansion histories.

Introduction – Over the past few years evidence for the accelerated expansion of the Universe has become overwhelming: besides supernovas [1, 2, 3, 4], every well-measured cosmological observable seems to support the conclusion that we no longer live in a decelerated, matter-dominated Einstein-de Sitter Universe [5, 6, 7, 8, 9]. The search for the cause of this acceleration, whether it is yet another dark component (dark energy) or if it is due to some modification of gravity which makes it repulsive at large scales [10, 11, 12, 13], has become one of the most pressing questions of our times.

The properties of dark energy, and its effect on cosmological observables, are commonly parametrized in terms of an equation of state $w_{de} = p_{de}/\rho_{de}$ with a simple, dependence on redshift, $w_{de}(z) = w_0 + w_a z/(1 + z)$ [14, 15]. Substantial observational efforts are underway [16, 17, 18], and many more are being planned [19, 20], in order to determine such parameters.

However, getting the equation of state even with excellent precision, in whatever parametrization, would still tell us close to nothing about the nature of dark energy [21], or if the acceleration is in fact due to modified gravity. One way of answering those questions is to look for evidence of dark energy perturbations in cosmological observables – evidence which must exist if dark energy is anything but the Cosmological Constant [22]. Notice that the signal of dark energy perturbations should not be confused with the influence of smooth dark energy on the growth function of dark matter inhomogeneities, which is another important, but distinct effect of dark energy upon structure formation [23, 24, 25].

Unfortunately, searching for imprints of dark energy fluctuations can sometimes be underwhelming: since dark energy dominates only at late times, its impact on the cosmic microwave background (CMB) arises only through the integrated Sachs-Wolfe effect, which in turn makes the signal both small, and limited to the largest scales where cosmic variance defeats the statistical purpose of the data [26, 27, 28, 29].

Dark energy fluctuations also affect the power spectrum $P(k)$ [21, 26, 27, 30, 31]. In particular, Hu [32] used an effective description for the dark energy pressure perturbations in order to estimate their impact on some cosmological observables. In that approximation, the pressure perturbation $\delta p_{de} = s_{\text{eff}} \delta\rho_{de}$, where s_{eff} is an *effective* sound speed squared and $\delta\rho_{de}$ is the energy density perturbation of dark energy [69]. This framework is not ideal, but it is extremely useful if our goal is to enlarge the class of models we want to constrain, just like any given parametrization of the equation of state is not meant to represent actual models, but to simplify the task of assessing the power of observations to constrain those models. Along these lines, e.g. Takada [33] regarded s_{eff} as a free parameter and concluded that measurements of the CMB and power spectrum could detect the effects of dark energy inhomogeneities, but only if the effective sound speed was very small, which would make it behave almost like dust on the scales probed by the CMB and $P(k)$.

That leaves the nonlinear regime of structure formation (collapsed structures, or halos) as one of the few venues left where we could expect the influence of dark energy to be measurable. As the initially overdense regions attract the surrounding matter and experience gravitational collapse, the steep gravitational potentials at the center of the dark matter halos deform the distribution of dark energy. And since pressure is a key ingredient in halo formation, dark energy could have an enhanced effect on the masses, times of formation and relative abundances of halos [34, 35, 36, 37, 38, 39, 40, 41, 42] as well as voids [43, 44]. Notice that both $P(k)$ and galaxy cluster counts are potentially sensitive tests of

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*Electronic address: abramo@fma.if.usp.br

dark energy perturbations. However, while the former tests the long-range, infra-red properties of the underlying theory of dark energy in the linear regime, the latter is a test of the small-scale, ultraviolet, nonlinear regime of the same unknown theory. They are, therefore, complementary rather than competing tests of the nature of dark energy – see also [45] on using cluster counts to constrain models of modified gravity.

Methodology – Counts of galaxy clusters have long been regarded as a crucial cosmological observable [46, 47], in particular for purposes of constraining dark energy models [48, 49, 50, 51, 52, 53, 54]. In the present work we study the sensitivity of cluster counts to dark energy perturbations. We employ a set of seven cosmological parameters that we allow to vary: h (the value of the Hubble expansion rate, where $H_0 = 100 \; h$ km s^{-1} Mpc^{-1}), Ω_m (the amount of matter relative to the critical density), Ω_b (density ratio of baryons), σ_8 (rms matter fluctuations on scales of 8 h^{-1} Mpc), w_0, w_a and $s_{\rm eff}$ (we assume flatness throughout.) The abundance of halos is then estimated with the Press-Schechter formalism [55], using the top-hat spherical collapse model [56] to compute the critical density at the time of collapse – see also [34, 46, 57, 58], and, in particular, [40, 42].

Our statistical analysis relies on the Fisher matrix, which was computed in this 7-dimensional parameter space for several hypothetical Sunyaev-Zel'dovich (SZ) and weak lensing (WL) surveys of galaxy clusters, and a suitable fiducial dynamical dark energy (DDE) model with $w_0 = -1.1$ and $w_a = 0.5$. We have checked that the impact of dark energy clustering scales like $|1+w|$, which is natural since in the ΛCDM limit there are no dark energy perturbations. The other fiducial values we used are $h = 0.72$, $\Omega_b = 0.05$, $\sigma_8 = 0.76$ and $\Omega_m = 0.25$. This model lies near the best-fit region in the parameter space of several recent joint analyses of cosmological observations [59, 60]. Notice that, even though our DDE model happens to experience "phantom crossing" ($w = -1$) at a redshift $z = 0.25$, there is no associated instability in the spherical collapse model, hence the sensitivity to dark energy is not artificially enhanced in this scenario compared to similar scenarios without phantom crossing – see, for instance, [61]. Finally, for completeness we studied three scenarios of DDE, with fiducial values $s_{\rm eff} = 0$, $s_{\rm eff} = 0.5$ and $s_{\rm eff} = -0.75$.

The main limitation to the sensitivity of number counts is the halo mass below which clusters cannot be detected with the given instrument and strategy. For the i-th bin the number of observed clusters is given in terms of the mass function as:

$$ N_i = \Delta\Omega \int_{z_i}^{z_{i+1}} dz \int_{M_i}^{M_{i+1}} dM \; \theta[M - M_{min}(z)]\frac{dn}{dMdz} \; , $$

where $\Delta\Omega$ is the solid angle subtended by the survey. The limiting mass M_{min} is approximately constant in redshift for surveys which employ the SZ effect, but in WL cluster surveys the limiting mass grows with redshift due to the
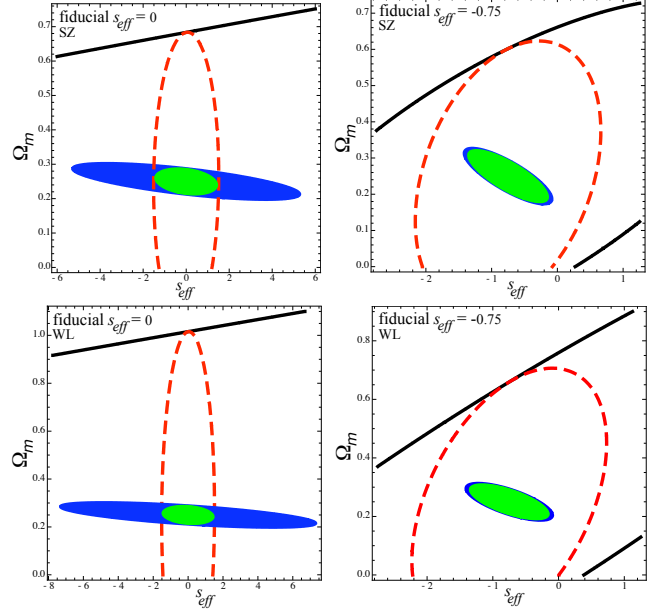

Figure 1: Joint Constraints on s_{eff} and Ω_m from near-future (nf) surveys. The top panels show marginalized 68% C.L. limits from SZ-type surveys, and the bottom panels show the limits from WL-type surveys, for the fiducial models with $s_{eff} = 0$ (left panels) and $s_{eff} = -0.75$ (right panels.) The thick solid (black in color version) lines correspond to cluster counts limits (68% C.L., marginalized); the thick dashed (red) lines assume an additional weak prior on s_{eff}, $\sigma^2_{s_{eff}} = 1$; the dark (blue) ellipses correspond to adding the "COSMO" set of priors (see text) to the cluster Fisher matrix; and the smaller, light (green) ellipses correspond to adding both the weak prior on s_{eff} and the COSMO priors.

declining number of background galaxies [62, 63].

We consider three hypothetical scenarios for these two types of surveys: near-future (nf) ones, such as the SPT/DES [16], which are supposed to cover about 4000 deg^2; future (f) ones, such as the LSST [18], which will cover about half the sky (18.000 deg^2); and far-future (ff) ones, such as EUCLID/JDEM [20, 64], which will cover up to 75% of the sky.

We have defined limiting masses by a simple two-parameter function: $\log_{10} M_{\rm min}/(h^{-1}M_\odot) = \lambda + \beta z$. We have set $\lambda_{nf}^{SZ} = 14.2$, $\lambda_{f}^{SZ} = 14.1$ and $\lambda_{ff}^{SZ} = 13.7$, and $\beta^{SZ} = 0$ in all cases – which means we assume exactly flat selection functions for the SZ-like surveys, clearly a very rough approximation. For the WL surveys we have set $\lambda_{\rm nf}^{WL} = 14.0$, $\lambda_{\rm f}^{WL} = 13.5$ and $\lambda_{\rm ff}^{WL} = 13.2$, and $\beta^{WL} = 0.6$ in all cases. With these limiting masses, we obtained (for ΛCDM) total numbers of clusters of: 7000 and 4600 respectively for the near-future SZ and WL surveys; ~ 280.000 for future surveys (both SZ and WL); and $\sim 1.5 \times 10^6$ for far-future surveys (both SZ and WL).

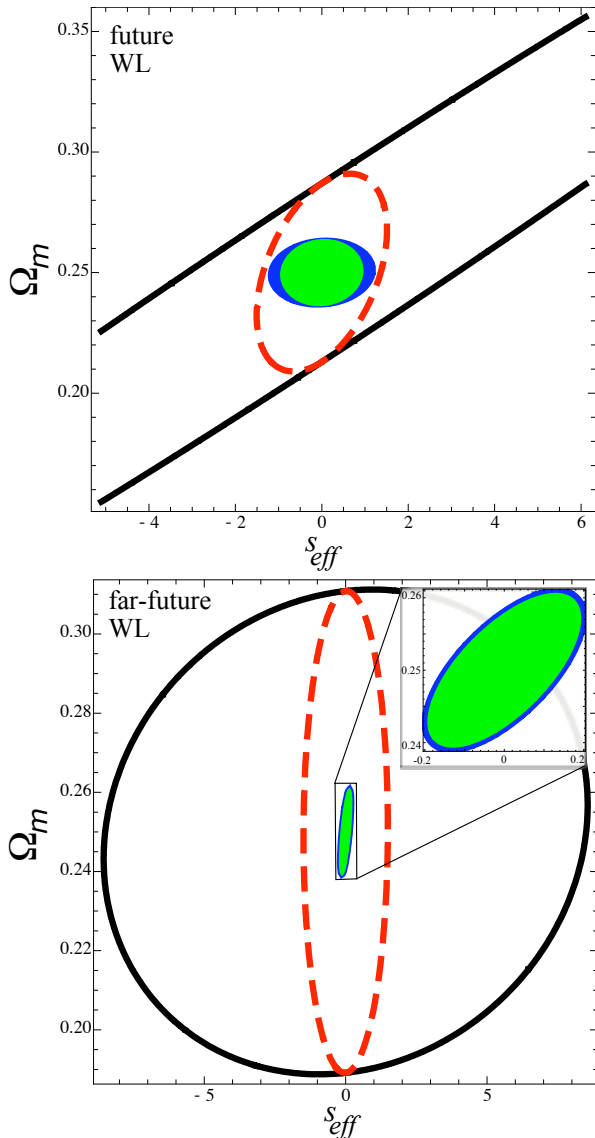We have used different binnings in mass and in redshift in each case: for near-future SZ and WL surveys

The Fisher matrix is then given by:

$$F_{ab} = \sum_i \frac{1}{N_i} \frac{\partial N_i}{\partial \theta^a} \frac{\partial N_i}{\partial \theta^b} ,$$

where θ^a are any one of the 7 cosmological parameters. If all other parameters are fixed, the 1σ (68% C.L.) limit on the parameter θ^a is given by $\sqrt{1/F_{aa}}$. If the other parameters are marginalized over (integrated out of the total PDF), then the 1σ limit on θ^a is given by $\sqrt{(F^{-1})_{aa}}$.

We have also considered the effect of adding an external ("COSMO") set of cosmological priors: WMAP 5y (through the shift parameter R) [67], BAO [6], HST [68] and a baryon fraction $\Omega_b h^2 = 0.02273 \pm 0.00062$, also from WMAP 5y data [67].

Results and Discussion – If the fiducial model is ΛCDM, then naturally the sensitivity to perturbations (and to the sound speed) of dark energy vanishes. For the dynamical dark energy fiducial model we find that the impact of the dark energy perturbations is dramatically different depending on the value of the dark energy pressure perturbation. If the pressure is positive, zero or negative, then the impact on the mass function (and on the number counts) is respectively very small, small or very large (see Figs. 1-3.) This means that the likelihood function is more peaked for smaller values of the sound speed.

For positive dark energy pressure, our results are similar to what would be expected from "quintessence" (canonical scalar field) models of dark energy, since in that case the Jeans length is approximately equal to the Compton wavelength of the dark energy field, $\lambda_J \simeq c_s/m_{de}$ ($c_s = \sqrt{s_{eff}}$ is the sound speed.) In that case, dark energy clusters only on horizon scales, and its effect on cluster counts is minute.

For near-vanishing dark energy pressure (sound speed very small) our results are similar to power-spectrum constraints like those by obtained by, e.g., Takada [33]. However, because the typical Jeans lengths are still typically much larger than cluster scales, the limits from number counts are weaker than the limits from $P(k)$.

For negative pressures ($s_{eff} < 0$) the sensitivity of galaxy cluster counts to dark energy perturbations increases dramatically. In fact, this would be true for the sensitivity of any large-scale structure observable (e.g., CMB and power spectrum), if it were not for the fact that it is probably nonsensical to consider negative sound speeds squared at the linear level. However, in collapsed regions and small scales there is nothing that forbids negative pressures [34, 42] – in fact, the bag model of hadronic physics is one microphysical example that supports the use and interpretation of negative pressure in cosmology.

Our main conclusions are:

1. Ongoing surveys such as the South Pole Telescope, which will detect a few thousands of clusters, are already sensitive enough to
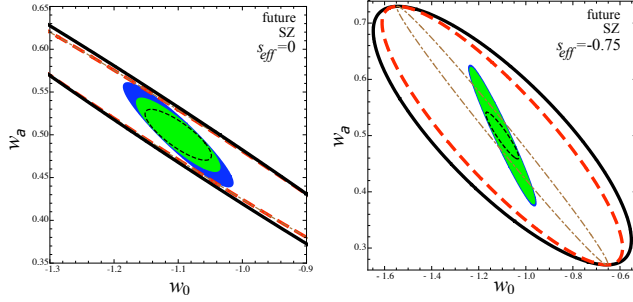

Figure 2: Joint Constraints on s_{eff} and Ω_m from WL surveys in the fiducial model with $s_{eff} = 0$, for future surveys (upper panel) and far-future surveys (lower.) The colors are the same as in Fig. 1.

we used 3 mass bins, and 10 redshift bins up to $z = 1.5$; for future SZ and WL surveys we used 5 mass bins and 15 redshift bins up to $z = 1.5$; and for the far-future SZ and WL surveys we used 8 mass bins and 25 redshift bins up to $z = 2.0$. Our results do not change significantly if fewer mass bins are used, and we chose fat redshift bins in order to minimize sample ("2-halo") variance [63, 65, 66]. In fact, we only consider Poisson (shot) noise in the covariance of the number of detected halos: $\langle (N_i - \bar{N}_i)(N_j - \bar{N}_j) \rangle = \delta_{ij} \bar{N}_i$, where N_i ($\bar{N}_i$) is the actual (expected) number of halos in the i-th bin.

Figure 3: Joint Constraints on w_0 and w_a from near-future SZ surveys, for the fiducial models with $s_{eff} = 0$ (left) and $s_{eff} = -0.75$ (right.) Legends for the thick solid and dashed lines, as well as for the ellipses, are identical to the previous figures. In addition, computing the 68% C.L. limits *without* the dark energy clustering parameter s_{eff} leads to either the thin dashed (black) or dot-dashed (brown) lines, whether or not one includes the COSMO set of priors.
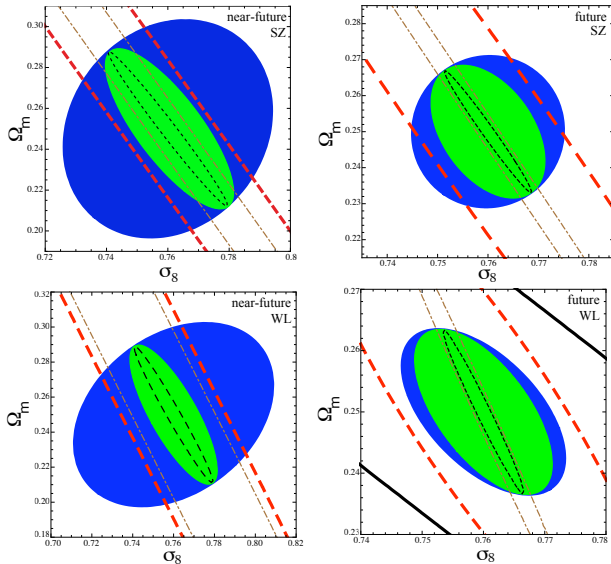

Figure 4: Joint Constraints on Ω_m and σ_8 from SZ (top) and WL (bottom), near-future (left) and future (right) surveys. In all panels the fiducial model has $s_{eff} = 0$. Legends are identical to Fig. 3

detect dark energy perturbations (in the absence of strong systematics) if $s_{eff} < 0$. Dark energy perturbations also weaken the

detection of the other dark energy parameters determined by the SPT, such as w_0 and w_a: in particular, the marginalized figure-of-merit $(1/\sqrt{\Delta w_0 \Delta w_a})$ is substantially smaller ($\sim 50\%$) compared to the case where dark energy perturbations are neglected. This points to a degeneracy between the effective sound speed and the equation of state which is not unexpected, since one determines the pressure on small scales, while the other sets the pressure on large scales.

2. Future surveys, such as the LSST, may or may not find evidence of dark energy perturbations. But even in the worst scenario ($s_{eff} > 0$, weakest dependence on the sound speed) the constraints on the main cluster parameters (such as Ω_m and σ_8) are substantially weakened by this additional "nuisance parameter". Furthermore, the marginalized limits on the equation of state parameters w_0 and w_a are affected at the level of 10-30%, depending on the scenario – see Figs. 3-4.

3. Surveys in the more distant future, such as ESA's Euclid or NASA's JDEM, will detect the imprints of the fluctuations of dark energy, but in the most conservative scenario the constraints would only be strong enough to determine whether the dark energy pressure in collapsed regions is positive or negative.

Therefore, if future data points to anything other than a Cosmological Constant, cluster surveys will be a key observation to determine the nature of dark energy. But even if cluster counts turn out to impose weak constraints on dark energy clustering, this would still be a minor ingredient affecting the halo mass function. Our results show that the addition variance (nuisance) introduced by dark energy clustering in the determination of the other cosmological parameters through galaxy cluster counts cannot be dismissed.

[1] Supernova Cosmology Project, S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999), astro-ph/9812133.

[2] Supernova Search Team, A. G. Riess *et al.*, Astron. J. **116**, 1009 (1998), astro-ph/9805201.

[3] Supernova Search Team, A. G. Riess *et al.*, Astrophys. J. **607**, 665 (2004), astro-ph/0402512.

[4] ESSENCE, W. M. Wood-Vasey *et al.*, Astrophys. J. **666**, 694 (2007), astro-ph/0701041.

[5] The SNLS, P. Astier *et al.*, Astron. Astrophys. **447**, 31 (2006), astro-ph/0510447.

[6] SDSS, D. J. Eisenstein *et al.*, Astrophys. J. **633**, 560 (2005), astro-ph/0501171.

[7] WMAP, D. N. Spergel *et al.*, Astrophys. J. Suppl. **170**, 377 (2007), astro-ph/0603449.

[8] Y. Wang and P. Mukherjee, Astrophys. J. **650**, 1 (2006), astro-ph/0604051.

[9] E. L. Wright, astro-ph/0701584.

[10] G. R. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. **B485**, 208 (2000), hep-th/0005016.

[11] S. M. Carroll, V. Duvvuri, M. Trodden, and M. S. Turner, Phys. Rev. **D70**, 043528 (2004), astro-ph/0306438.

[12] S. M. Carroll *et al.*, Phys. Rev. **D71**, 063513 (2005), astro-ph/0410031.

[13] B. Jain and P. Zhang, (2007), 0709.2375.

[14] M. Chevallier and D. Polarski, Int. J. Mod. Phys. **D10**, 213 (2001), gr-qc/0009008.

[15] A. Albrecht *et al.*, (2006), astro-ph/0609591.

[16] The SPT, J. E. Ruhl *et al.*, Proc. SPIE Int. Soc. Opt. Eng. **5498**, 11 (2004), astro-ph/0411122.

[17] Dark Energy Survey, T. Abbott *et al.*, (2005), astro-ph/0510346.

[18] LSST, Z. Ivezic *et al.*, (2008), 0805.2366.

[19] SNAP, G. Aldering *et al.*, (2004), astro-ph/0405232.

[20] DUNE, A. Refregier *et al.*, (2006), astro-ph/0610062.

[21] J.-P. Uzan, Gen. Rel. Grav. **39**, 307 (2007), astro-ph/0605313.

[22] K. Coble, S. Dodelson, and J. A. Frieman, Phys. Rev. **D55**, 1851 (1997), astro-ph/9608122.

[23] E. V. Linder and A. Jenkins, Mon. Not. Roy. Astron. Soc. **346**, 573 (2003), astro-ph/0305286.

[24] E. V. Linder, Phys. Rev. **D72**, 043529 (2005), astro-ph/0507263.

[25] G. Ballesteros and A. Riotto, Phys. Lett. **B668**, 171 (2008), 0807.3343.

[26] J. K. Erickson, R. R. Caldwell, P. J. Steinhardt, C. Armendariz-Picon, and V. F. Mukhanov, Phys. Rev. Lett. **88**, 121301 (2002), astro-ph/0112438.

[27] S. DeDeo, R. R. Caldwell, and P. J. Steinhardt, Phys. Rev. **D67**, 103509 (2003), astro-ph/0301284.

[28] L. R. Abramo, F. Finelli, and T. S. Pereira, Phys. Rev. **D70**, 063517 (2004), astro-ph/0405041.

[29] W. Hu and R. Scranton, Phys. Rev. **D70**, 123002 (2004), astro-ph/0408456.

[30] R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998), astro-ph/9708069.

[31] T. Chiba, N. Sugiyama, and T. Nakamura, Mon. Not. Roy. Astron. Soc. **301**, 72 (1998), astro-ph/9806332.

[32] W. Hu, Phys. Rev. **D65**, 023003 (2002), astro-ph/0108090.

[33] M. Takada, Phys. Rev. **D74**, 043505 (2006), astro-ph/0606533.

[34] D. F. Mota and C. van de Bruck, Astron. Astrophys. **421**, 71 (2004), astro-ph/0401504.

[35] N. J. Nunes and D. F. Mota, Mon. Not. Roy. Astron. Soc. **368**, 751 (2006), astro-ph/0409481.

[36] M. Manera and D. F. Mota, Mon. Not. Roy. Astron. Soc. **371**, 1373 (2006), astro-ph/0504519.

[37] N. J. Nunes, A. C. da Silva, and N. Aghanim, Astron. Astroph. **450**, 899 (2005), astro-ph/0506043.

[38] I. Maor and O. Lahav, JCAP **0507**, 003 (2005), astro-ph/0505308.

[39] L. Liberato and R. Rosenfeld, JCAP **0607**, 009 (2006), astro-ph/0604071.

[40] L. R. Abramo, R. C. Batista, L. Liberato, and R. Rosen-feld, JCAP **0711**, 012 (2007), arXiv:0707.2882 [astro-ph].

[41] L. R. W. Abramo, R. C. Batista, L. Liberato, and R. Rosenfeld, Phys. Rev. **D77**, 067301 (2008), arXiv:0710.2368 [astro-ph].

[42] L. R. Abramo, R. C. Batista, L. Liberato, and R. Rosenfeld, Phys. Rev. **D79**, 023516 (2008), 0806.3461.

[43] S. Dutta and I. Maor, Phys. Rev. **D75**, 063507 (2007), gr-qc/0612027.

[44] D. F. Mota, D. J. Shaw, and J. Silk, (2007), arXiv:0709.2227 [astro-ph].

[45] F. Schmidt, M. V. Lima, H. Oyaizu, and W. Hu, (2008), 0812.0545.

[46] P. T. P. Viana and A. R. Liddle, Mon. Not. Roy. Astron. Soc. **281**, 323 (1996), astro-ph/9511007.

[47] N. A. Bahcall, X. Fan, and R. Cen, Astrophys. J. **485**, L53 (1997), astro-ph/9706018.

[48] L.-M. Wang and P. J. Steinhardt, Astrophys. J. **508**, 483 (1998), astro-ph/9804015.

[49] Z. Haiman, J. J. Mohr, and G. P. Holder, Astrophys. J. **553**, 545 (2000), astro-ph/0002336.

[50] J. Weller, R. Battye, and R. Kneissl, Phys. Rev. Lett. **88**, 231301 (2002), astro-ph/0110353.

[51] R. A. Battye and J. Weller, Phys. Rev. **D68**, 083506 (2003), astro-ph/0305568.

[52] S. Basilakos, Astrophys. J. **590**, 636 (2003), astro-ph/0303112.

[53] E. L. Lokas, P. Bode, and Y. Hoffman, Mon. Not. Roy. Astron. Soc. **349**, 595 (2004), astro-ph/0309485.

[54] S. Wang, J. Khoury, Z. Haiman, and M. May, Phys. Rev. **D70**, 123008 (2004), astro-ph/0406331.

[55] W. H. Press and P. Schechter, Astrophys. J. **187**, 425 (1974).

[56] J. E. Gunn and I. Gott, J. Richard, Astrophys. J. **176**, 1 (1972).

[57] P. Fosalba and E. Gaztanaga, Mon. Not. Roy. Astron. Soc. **301**, 503 (1998), astro-ph/9712095.

[58] W. J. Percival, Mon. Not. Roy. Astron. Soc. **327**, 1313 (2001), astro-ph/0107437.

[59] Y. Wang, Phys. Rev. **D77**, 123525 (2008), 0803.4295.

[60] A. Vikhlinin *et al.*, (2008), 0812.2720.

[61] W. Hu, Phys. Rev. **D71**, 047301 (2005), astro-ph/0410680.

[62] S. Majumdar and J. J. Mohr, Astrophys. J. **613**, 41 (2004), astro-ph/0305341.

[63] L. Marian and G. M. Bernstein, Phys. Rev. **D73**, 123525 (2006), astro-ph/0605746.

[64] A. J. Albrecht *et al.*, (2009), 0901.0721.

[65] W. Hu and A. V. Kravtsov, Astrophys. J. **584**, 702 (2003), astro-ph/0203169.

[66] M. Takada and S. Bridle, New J. Phys. **9**, 446 (2007), 0705.0163.

[67] WMAP, J. Dunkley *et al.*, Astrophys. J. Supp. **180**, 306 (2009), 0803.0586.

[68] HST, W. L. Freedman *et al.*, Astrophys. J. **553**, 47 (2001), astro-ph/0012376.

[69] This is *not* an approximation only if one works in the comoving gauge [32], in which case obviously the Einstein equations inherit the gauge terms.